Advanced Numerical Methods for Physics (3 CFU)

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Program of the course:

General introduction to Partial Differential Equations (PDEs). Classification of 2nd order PDEs and canonical form thereof. Reduction to nondimensional form. Parabolic equations. Diffusion equation. Explicit and implicit methods. Stability. Truncation error and local truncation error. Consistence or compatibility. Refinement path. Discretization error: convergence. Reminder of vector and matrix norms. Consistent and subordinate norms for matrices: p-norms. Spectral radius. Gerschgorin's theorems. Convergence, compatibility, stability (Lax-Richtmyer). Examples. Unconditional stability of the Crank-Nicolson scheme. Analytical solution of (separable) PDEs by Fourier series. Stability again: Fourier analysis and stability according to von Neumann. Examples. Lax equivalence theorem. Hyperbolic equations. Advection equation. Derivation from the continuity equation in integral form. Characteristic lines. Quasi-linear equations and shock-waves (breaking). Burger's equation. Numerical integration of the advection along a characteristic line. Upwind method. Courant-Friedrichs-Lewy (CFL) condition. Second-order quasi-linear hyperbolic equations. Characteristic lines. D'Alembert solution of the wave equation. Numerical domain of dependence of the light-cone.

Additional topics:

Elliptic equations. Poisson and Laplace equations. The biharmonic equation and its associated eigenvalue problem. Iterative and direct numerical methods (finite differences). Finite elements methods: overview. Numerical methods for the 1+1 dimensional Schrödinger equation. Nonlinear equations: exact and numerical methods. Transport phenomena in solids: Boltzmann equation.

Bibliography:

Scientific papers and slides provided by the teacher